

Pipe Network Analysis for Solar Thermal Plants

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Abstract

Efficiency, cost-effectiveness and operational safety of solar plants must be based on the knowledge of pressure loss as well as flow distribution and temperature distribution of branched collector arrays. Therefore, pipe network analysis is an essential step during the dimensioning process. This article presents an explicit, fast converging numerical scheme especially suited for collector fields. The effect of pipe dimensions and pipe routing on flow, temperature and pressure-distribution and collector field efficiency is demonstrated by application of a dedicated software tool, which is provided as an open source code.

Keywords: Pipe network analysis, flow distribution, pressure loss

1. Introduction

Circuits of large solar plants are branched networks of pipes. Pipe network analysis is needed to find the best solution among the many possible ways of pipe routing, the corresponding pipe diameters and the pump size. The optimum is characterized by the minimum of costs while each of the auxiliary conditions, e.g. pressure loss, limiting flow velocity, maximum variation of exit temperatures and thermal efficiency, are fulfilled. Several approaches for cost optimization of pipe networks can be found in literature. Murphy et al. (1993) developed an algorithm for water supply networks. Frank (2007) derived a numerical scheme for the optimization of pipe routing, pipe dimension and pump power of a large solar plant for district heating. Park et al. (2017) developed a model for the optimization of district heating networks.

A pipe network is characterized by the geometry of the pipes and the nodes where pipes are connected to other pipes. A node connecting two pipes is called junction. A node where two pipes with the same direction are connected and a third pipe is connected perpendicularly to the same node is called T-junction. A node is considered zero-dimensional. It can therefore store neither mass nor energy. A pipe or a series of pipes connecting two neighboring nodes defines a branch. A branch can also contain hydraulic elements such as valves, pumps or heat exchangers. A path consists of a series of branches. A closed path within a pipe network defines a loop. A loop is called elementary if the number of its nodes cannot be reduced by a shortcut. A pipe network is irregular if elementary loops of different size are arranged in an irregular pattern. A network is called periodical if it is dominated by a repeated arrangement of identical elementary loops. Periodical networks usually contain branches that dominate the pressure loss. These branches are named strings. Strings often consist of a series of identical hydraulic elements, e.g. serially connected solar collectors. In periodical networks, arrays of identical strings are connected in parallel by distribution and collection manifolds.

All methods for pipe network analysis are based on conservation equations for mass and momentum. Since fluid properties are temperature dependent, energy conservation must also be considered. The flow distribution within a pipe network is expressed by two sets of equations. The node-equations (1) define mass conservation in each node, j , where, m is the number of pipes connected, ρ_k is the density of the liquid entering or leaving the branch, k , and, \dot{V}_k is the flow rate in the same branch.

$$\sum_{k=1}^m \rho_k \dot{V}_k = 0 \quad (1)$$

The loop-equations (2) formulate momentum conservation. In steady state operation the sum of pressure losses, Δp_k , of the branches, k , of any closed loop is zero.

$$\sum_{k=1}^n \Delta p_k = 0 \quad (2)$$

In section 2 of this article, an equivalent formulation is used: The sum of pressure losses between two nodes, e.g. inlet and outlet of a collector field, is the same for all paths. Because of the non-linear relationship between flowrate and pressure loss these equations are solved by iterative methods. The choice of method depends primarily on the structure of the network, the number of nodes and the mode of operation. In the following, only methods for steady state flow are considered.

The probably oldest and still widely used method was developed by Cross (1936). The iterative procedure starts with initial values for the mass flow, which satisfy the node-equations. The evaluation of the loop-equation results in an error that is used to determine a correction of the flow rate, ΔV , which is then added to the flow of all branches in the loop. The loop equation is subsequently applied to each elementary loop of the network. Derivation of Eq. (3) can be found in many textbooks, e.g. Massoud (2005), Horlacher and Lüdecke (2006) and Eismann (2017).

$$\Delta \dot{V}_k = - \frac{\sum_{i=1}^m R_{k,i} \dot{V}_{k,i} |\dot{V}_{k,i}|}{2 \sum_{i=1}^m R_{k,i} |\dot{V}_{k,i}|} \quad (3)$$

In Eq. (3) the elementary mesh, k , has m branches. The term $R_{k,i}$ is the dimensional friction factor,

$$R_{k,i} = \lambda_{k,i} \frac{8 l_{k,i} \rho_{k,i}}{d_{k,i}^5 \pi^2}, \quad (4)$$

which is a function of a dimensionless friction factor, λ , the density, ρ , of the fluid and the pipe dimensions, l and d . The procedure is repeated until the sum of relative changes of the flowrates is below a predefined margin. The method of Cross (1936) represents the class of explicit, sequential loop-methods. It is applicable for both irregular and periodical networks. Because of the sequential procedure, convergence is rather slow.

2. Method for periodical networks with dominant strings

Networks of solar thermal plants are essentially periodical. Figure 1 shows an array of n strings connected in parallel by distribution and collection manifolds. The strings, S_k , $k = 1..n$, consist of one or more serially connected absorbers. The manifolds consist of a series of identical distribution pipes, D_k , and collection pipes, C_k . There are two ways of connecting arrays of strings. One-sided connection as shown in Figure 1 a) is figuratively called C-

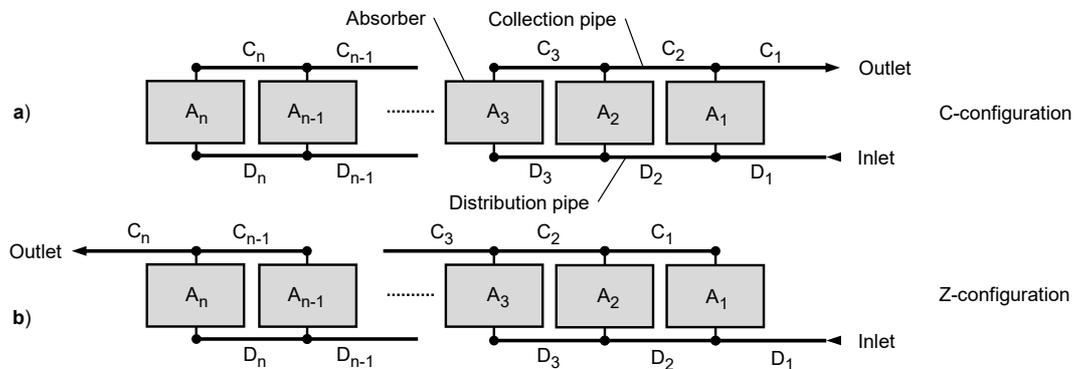


Figure 1 Row of Absorbers connected in parallel a) C-configuration and b) Z-configuration.

configuration because the flow path between inlet and outlet through any of the strings resembles the letter C. Figure 1 b) shows the alternative Z-configuration where all flow paths between inlet and outlet have the same length.

The primary goal of any method of pipe network analysis is to find the steady state mass flow distribution in an array of strings for a predetermined total mass flow, \dot{m} . Based on the corresponding chapter in Eismann (2017, p. 37) a fast converging method, well adapted for arrangements shown in Figure 1, is derived as follows.

The dominance of the strings should be expressed by a dimensionless number valid for the whole array. The obvious choice is the ratio of pressure losses of the strings and the corresponding distribution and collection pipe expressed by the Darcy-Weisbach (Brown 2002) formula,

$$\Delta p_{S,k} = \lambda_k \frac{l_S}{d_S} \frac{2}{\rho_{S,k}} w_{S,k}^2 . \quad (5)$$

It is therefore necessary to represent strings, which can have a rather complex internal hydraulic circuit, by an approximately equivalent pipe with a length, l_S , and a diameter, d_S . Setting friction factors and flow velocities to the same values results in an expression which can be interpreted as a dominance ratio,

$$R_D = \frac{l_S d_D}{2 d_S l_D} . \quad (6)$$

The iterative procedure is initiated by a uniform mass flow distribution.

$$\dot{m}_{S,k} = \dot{m}/n \quad (7)$$

In a first step, mass conservation is applied to each node to calculate the mass flows in the distribution and collection pipes. The mass flows for C-configuration are,

$$\dot{m}_{D,k} = \dot{m}_{C,k} = \sum_{i=k}^n \dot{m}_{S,i} , \quad (8)$$

and for Z-configuration,

$$\dot{m}_{D,k} = \sum_{i=k}^n \dot{m}_{S,i} \quad ; \quad \dot{m}_{C,k} = \sum_{i=1}^k \dot{m}_{S,i} . \quad (9)$$

In a second step, the flow velocities in all distribution pipes, collection pipes and strings, $w = 4\dot{m}/\pi d^2 \rho$, and the pressure losses along each path between inlet and outlet are calculated. The pressure losses for C- configuration are,

$$\Delta p_k = \sum_{i=1}^k \Delta p_{D,i} + \Delta p_{S,k} + \sum_{i=1}^k \Delta p_{C,i} , \quad (10)$$

and for Z-configuration,

$$\Delta p_k = \sum_{i=1}^k \Delta p_{D,i} + \Delta p_{S,k} + \sum_{i=k}^n \Delta p_{C,i} . \quad (11)$$

Furthermore, the average pressure loss of the paths between inlet and outlet is calculated.

$$\langle \Delta p \rangle = \frac{1}{n} \sum_{k=1}^n \Delta p_k \quad (12)$$

Pipe friction is usually, by far, the dominant contribution to the pressure loss. Each term can incorporate minor losses due to elbows, abrupt change of diameter and T-junctions. Momentum conservation requires the pressure losses calculated by Eq. (10) or Eq. (11), respectively, to be equal. With the initial uniform mass flow distribution, however, the pressure losses will differ significantly.

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The following third step, which corrects the mass flow rates, is the key to the method. It is based on the following considerations. The friction factor for laminar flow in cylindrical tubes,

$$\lambda_{lam} = \frac{64}{Re} \quad \text{with} \quad Re = \frac{wd}{\nu} , \quad (13)$$

results in a pressure loss proportional to the mass flow. The friction factor of Blasius (1913) for turbulent flow in hydraulically smooth pipes,

$$\lambda_{turb} = 0.3451 \cdot Re^{-0.25} , \quad (14)$$

results in a pressure loss proportional to $\dot{m}^{1.75}$. The mass flow of each string is simultaneously corrected such that the pressure loss of the corresponding path equals the average pressure loss.

$$\dot{m}'_{S,k} = \dot{m}_{S,k} \left(\frac{\langle \Delta p \rangle}{\Delta p_k} \right)^{\frac{\gamma}{f}} \quad (15)$$

The transition region where both laminar and turbulent flow can occur is defined as $2200 < Re < 3000$. The denominator, f , of the exponent in Eq. (15) is,

$$f = \begin{cases} 1.75 & ; \quad \min(Re_{S,k}) > 2200 \\ 1 & ; \quad \max(Re_{S,k}) \leq 2200 \end{cases} . \quad (16)$$

Convergence can be enhanced by an over-relaxation factor, $\gamma > 1$, whose optimum value depends on the dominance ratio. If the Reynolds number of any string is within the transition region, an over-relaxation factor of one is recommended.

Because of the nonlinear relationship between mass flow and pressure loss, the sum of the new mass flows is different from the predetermined total mass flow, \dot{m} . In a fourth and final step, each mass flow is correct by the ratio of the predetermined total mass flow and the iterative solution of the total mass flow. The new approximation for the mass flows are,

$$\dot{m}''_{S,k} = \dot{m}'_{S,k} \frac{\dot{m}}{\sum \dot{m}'_{S,k}} . \quad (17)$$

Steps 1 to 4 are repeated until the approximate solution is close enough to the theoretical limit. A suitable criterion is the relative standard deviation of the pressure losses, whose limit, σ , can be set according to the required accuracy, e.g. $\sigma = 0.001$.

$$\sigma > \frac{1}{\langle \Delta p \rangle} \sqrt{\sum \frac{(\Delta p_k - \langle \Delta p \rangle)^2}{n-1}} \quad (18)$$

For dominance ratios above $R_D = 2.5$ the method converges faster than the method of Cross (1936). It is also applicable to hierarchically structured, periodical networks, where periodically arranged collector arrays are regarded as strings. However, requirement on periodicity can be alleviated considerably. The method is practical in cases where arrays have different numbers of strings and/or where the manifolds consist of a series of pipes with different diameters and length.

The following considerations are essential for any numerical method for pipe network analysis. Reynolds numbers in solar plants are quite low and often cover the regions of laminar-, transitional- and turbulent flow. Discontinuity at the transition between laminar and turbulent flow cause convergence problems. It is therefore mandatory to use friction factor correlations which are continuous functions of the Reynolds number covering laminar, transitional, and turbulent flow regions (Eismann and Adams 2018). In the following examples, the correlations (19) to (21) of Zanke (1993) are used, adapted for a transition region $2200 < Re < 3000$. The probability for turbulent flow in the transition region is,

$$P_{turb} = \exp[-\exp(10.45 - 0.0043 Re)] = 1 - P_{lam} . \quad (19)$$

The friction factor, Eq. (21), is defined as a linear combination of the laminar friction factor, Eq. (13), and the (explicit!) turbulent friction factor, Eq. (20),

$$\lambda_{turb} = \left[-2 \log_{10} \left(\frac{2.7 (\log_{10} Re)^{1.2}}{Re} + \frac{\varepsilon}{3.71 \cdot d} \right) \right]^{-2}, \quad (20)$$

weighted by their respective probabilities:

$$\lambda = (1 - P_{turb}) \lambda_{lam} + P_{turb} \lambda_{turb} \quad (21)$$

3. Examples

3.1. Convergence properties

Convergence of the method is demonstrated using the example of an array of 10 strings. Each string consists of one flat plate collectors with 2.3 m² aperture area and a meander-type absorber. The absorber tube is characterized by an equivalent pipe length of 18 m and 7 mm inner diameter. The strings are connected in C-configuration by distribution and collection pipes of 2.2 m length and 16 mm inner diameter. Minor losses in T-junctions are neglected. The flow is isothermal. Fluid properties of water at 20 °C, $\nu = 1.044 \cdot 10^{-6}$ m²/s and $\rho = 998$ kg/m³, are used. The dominance ratio is $R_D = 10.3$. The accuracy limit was set to $\sigma = 0.001$.

The total mass flow is 0.192 kg/s, which corresponds to an area specific volumetric flow rate of 30 l/hm². The Reynolds number in all strings is above $Re = 3000$. According to Eq. (16), the appropriate C. Figure 2 shows iterative solutions of the flow distribution. Both the new method and the method of Cross (1936) converge towards the same final solution. With an over-relaxation factor of 1.1 the new method reaches the solution after three iterations, whereas the method of Cross (1936) requires 36 iterations.

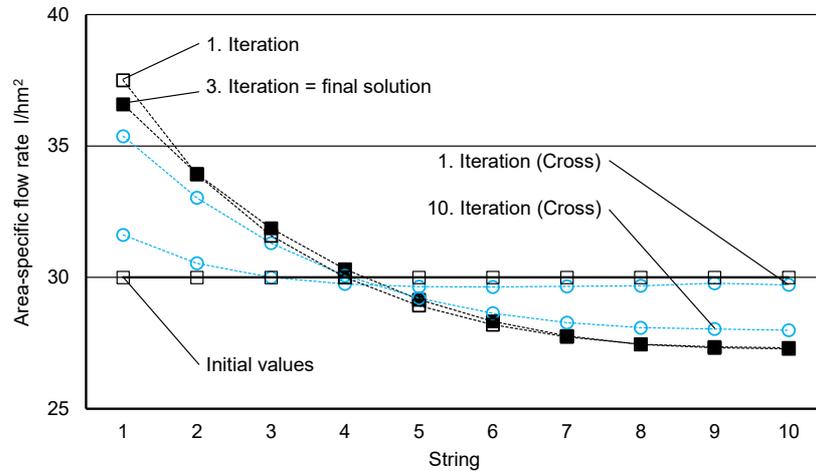


Figure 2 Convergence from initial uniform flow distribution of 30 l/hm² to the final solution.

For an area specific volumetric flow rate of 10 l/hm² the Reynolds number in all strings is below $Re = 2200$. With an over-relaxation factor of 1.2 the new method reaches the solution after four iterations, whereas the method of Cross (1936) requires 81 iterations. The reason for the fast convergence for purely laminar or turbulent flow lies in the denominator, f , which corresponds perfectly with the flow pattern of all strings.

Figure 3 shows the results for an average area specific volumetric flow rate of 20 l/hm². The Reynolds numbers $1857 \leq Re \leq 2830$ cover parts of the laminar and transition region. Because there is a probability for turbulent flow the denominator, f , must be set to the value 1.75. Application of Eq. (15) to strings with laminar flow results in slower convergence. The new method reaches the solution with 11 iterations compared to 46 iterations of the method of Cross.

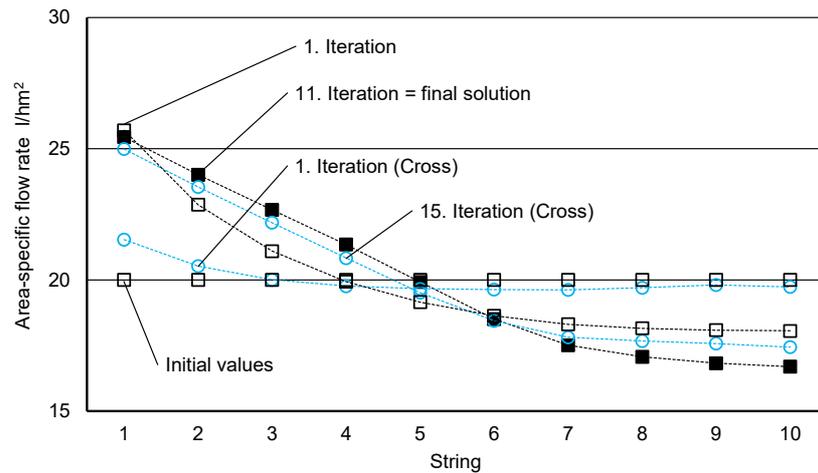


Figure 3 Convergence from initial uniform flow distribution of 20 l/hm² to the final solution.

3.2 Practical example

A practical example in two versions is discussed using the EXCEL/VBA tool HYDRA dedicated to the dimensioning and thorough cost optimization of solar circuits. It is based on the above-described method for pipe network analysis and includes models for solar collectors, cylindrical pipes and corrugated metal hoses, pumps, check valves, fittings and heat exchangers. The following example illustrates some of the capabilities of the software.

A collector field consists of six arrays of glazed flat plate collectors with selective absorbers arranged in a row with a distance of 4 m between the leading edges of the arrays. Each collector has an absorber as specified in section 3.1. The conversion factor and the heat loss coefficients are, $\eta_0 = 0.8$ -, $a_1 = 3.6$ W/Km² and $a_2 = 0.01$ W/K²m². Distribution and collection pipes have an inner diameter of 16 mm and a length of 2 m. Solar gain is transferred to the liquid by the absorber tubes only. Metal bellows arranged between adjacent pipe ends compensate thermal expansion of the distribution and collection pipes. Metal bellows are characterized by a hydraulic diameter of 16 mm, a length of 60 mm and a constant turbulent friction factor of $\lambda = 0.095$. The arrays are connected in parallel by staged field pipes with an inner diameter of 25, 32 and 39 mm. For the pressure losses in T-junctions the correlations of Wagner (2001) are used. Fluid properties of the water-glycol mixture TYFOCOR[®]LS[®] (TYFOROP 2015) are used. The area-specific flow rate is 30 l/hm². Inlet temperature is 55 °C. Solar irradiation is 1000 W/m² and ambient temperature is 20 °C. Figure 4 shows an arrangement where both the collectors and the arrays are connected in Z-configuration.

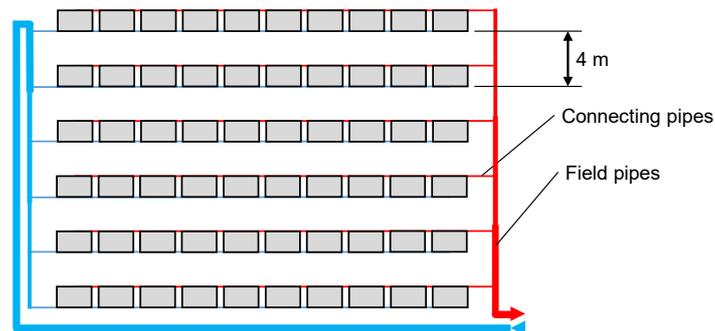


Figure 4 Flat-plate collector field. Collectors and arrays connected in Z-configuration.

Figure 5 shows the flow distribution of the collector field. As can be expected, the inhomogeneity of the flow distribution is nearly symmetrical and rather small. The temperature at the outlet is 74 °C and the useful gain is 84.4 kW. The pressure loss between inlet and outlet is 37.5 kPa.

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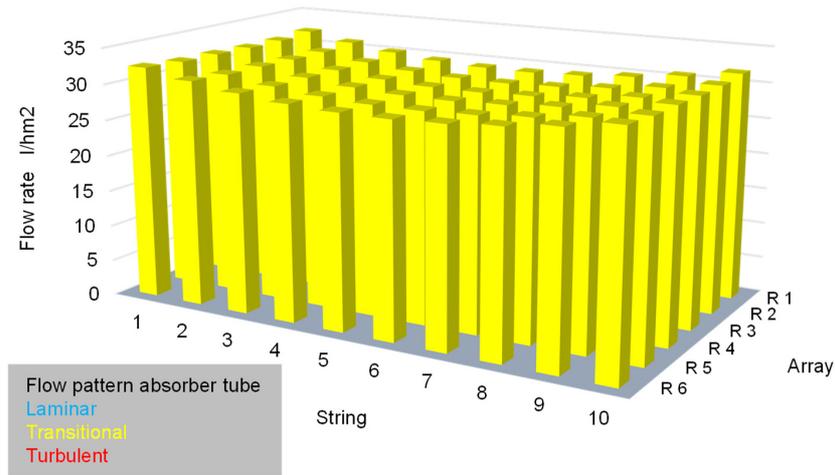


Figure 5 Flow distribution. Collectors and arrays connected in Z-configuration.

In the version shown in Figure 6, both the collectors and the arrays are connected in C-configuration. The flow distribution shown in Figure 7 is more inhomogeneous than with Z-configuration.

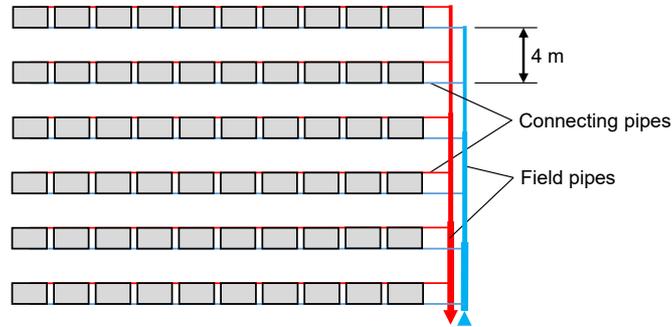


Figure 6 Flat-plate collector field. Collectors and arrays connected in C-configuration.

Expectedly, the highest flowrate occurs in the first string of the first array whereas the flow rate in the last string of the sixth array is smallest. The flow patterns cover the complete transitional region (yellow bars) and the adjacent regions of laminar (blue bars) and turbulent flow (red bars).

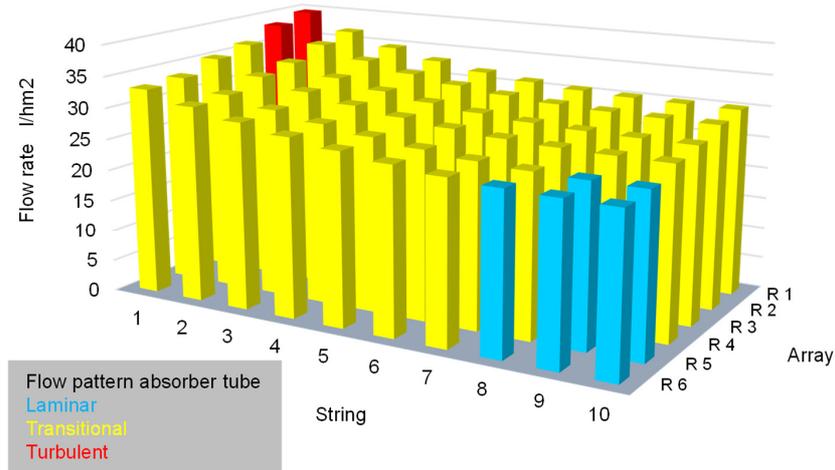


Figure 7 Flow distribution. Collectors and arrays connected in C-configuration.

C-configuration is much more economic than Z-configuration. The costs of about 40 meters of field pipes is saved. Because of the smaller pressure loss of only 30 kPa the operational costs are also lower. Furthermore, less space for the pipe routing is needed.

In all operational states, the pressure must be well above atmospheric pressure and vapor pressure. Therefore, information about pressure and temperature distributions is needed. Figure 8 shows the pressure in the distribution and collection manifolds.

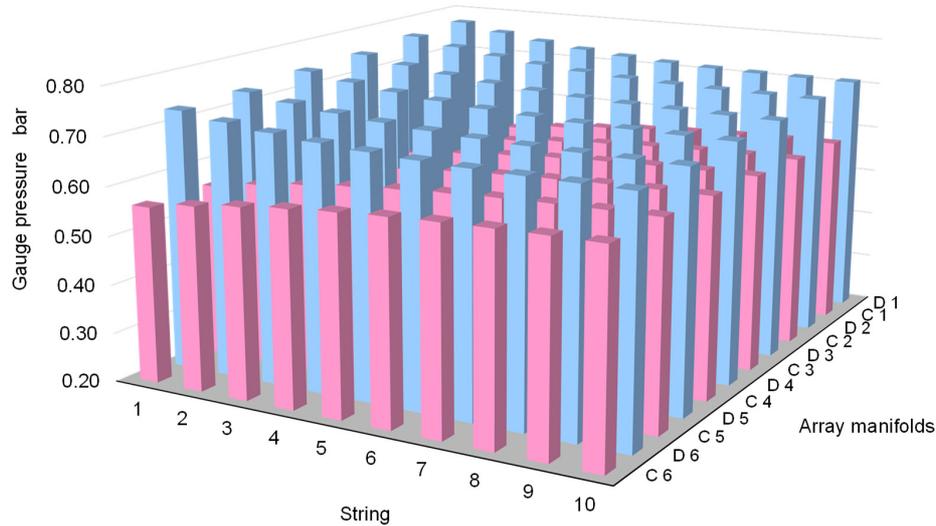


Figure 8 Pressure distribution. Collectors and arrays connected in Z-configuration.

The more pronounced inhomogeneity of the flow distribution causes a corresponding distribution of the exit temperature shown in Figure 9. However, the loss of collector efficiency compared to Z-configuration is below 0.2 % and easily compensated by the avoided heat loss of 40 m field pipes. The arrays are arranged in reverse order for better visibility. The base plane of the diagram at 55 °C indicates the temperature of the entering fluid.

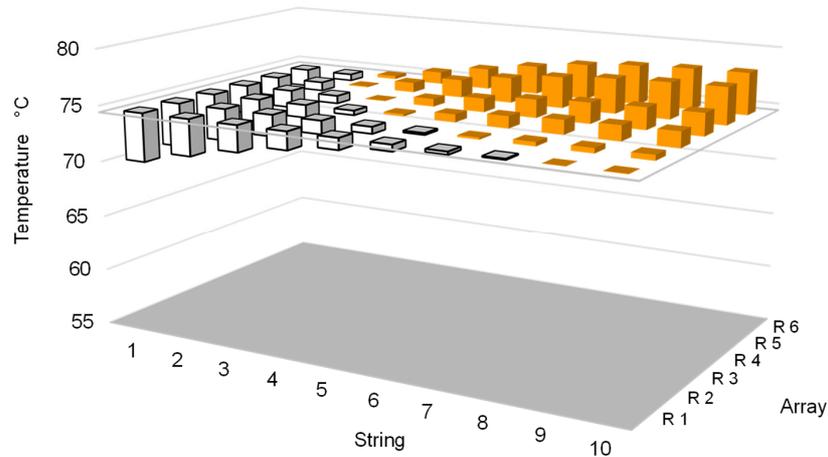


Figure 9 Exit temperature distribution. Collectors and arrays connected in C-configuration.

Figure 10 shows the temperature distribution in the header tubes. If the temperature sensor is placed in the collection pipe, the first collector of the third array should be chosen where the temperature differs only little from the temperature in the forward line.

In order to optimize venting capacity and to prevent partial stagnation, stagnant pipe sections must be avoided. Pipe routing is therefore predetermined by the collector hydraulics. Examples for flat plate collectors are shown in Figure 11.

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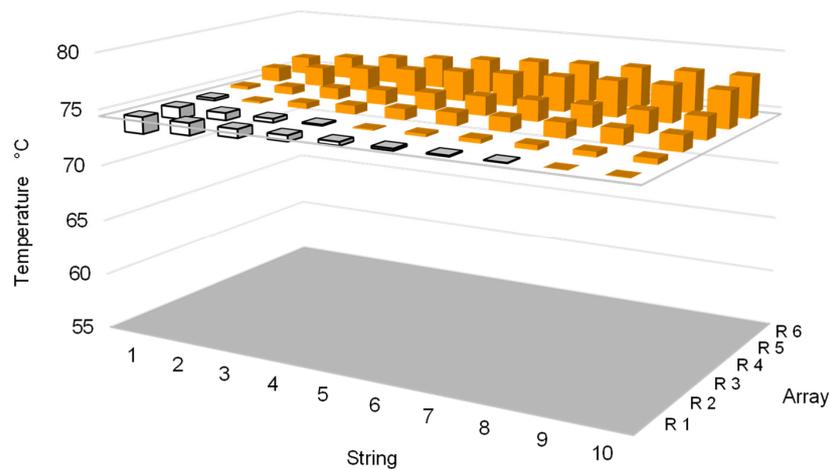


Figure 10 Temperature distribution in the collection manifolds. Collectors and arrays connected in C-configuration.

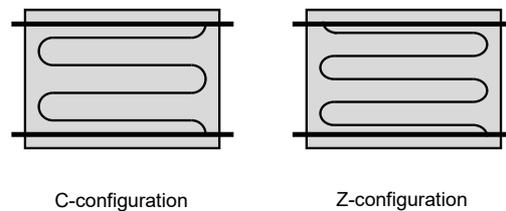


Figure 11 Flat-plate collectors suited for C- and Z-configuration

4. Conclusions

A method for pipe network analysis was derived, suitable for periodical networks with dominant strings. The method is not only explicit and therefore easy to program but also simultaneous which accounts for the fast convergence. For dominance ratios $R_D > 10$, which is a typical value for a flat plate collector with meander-type absorber, convergence is of the magnitude 10 times faster than the explicit, sequential method of Cross (1936).

The Excel/VBA tool HYDRA, dedicated to the dimensioning and thorough cost optimization of solar circuits, was used to demonstrate the application of the method. Comparison of C- and Z-configurations of a large collector field has shown that the influence of inhomogeneous flow distribution on the efficiency is negligible. In case of the example, one would choose the much cheaper C-configuration. However, it is not advisable to generalize this conclusion. Other aspects, like the internal collector hydraulics, venting capability and the flow velocity limit must also be considered. HYDRA is applicable to the design of solar thermal plants with glazed and unglazed flat plate collectors, vacuum tube collectors and PV/T collectors.

The author provides HYDRA as an open source code (Eismann 2018), with the objective of supporting the solar thermal market.

5. Acknowledgements

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6. Nomenclature

A	Collector area	w	average flow velocity [m/s]
a_1	linear heat loss coefficient [W/Km ²]	<i>Greek Symbols</i>	
a_2	quadratic heat loss coefficient [W/K ² m ²]	γ	
d	pipe diameter [m]	η_0	conversion factor [-]
l	pipe length [m]	λ	friction factor [-]
\dot{m}	mass flow rate [kg/s]	ν	kinematic viscosity [m ² /s]
P_{lam}	probability for laminar flow [-]	ρ	density [kg/m ³]
P_{turb}	probability for turbulent flow [-]	<i>Subscripts</i>	
Δp	pressure loss [bar, Pa]	C	collection pipe
R_D	dominancy ratio [-]	D	distribution pipe
Re	Reynolds number [-]	S	string
\dot{V}	volumetric flow rate [m ³ /s]		

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